KVS BHOPAL REGION CBT TEST OCT 2023
SUBJECT-MATHEMATICS
CLASS-12

TOPICS:

DIFFERENTIAL EQUATIONS, VECTORS

Case Study 1

If the equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then the solution of the differential equation is given by $ye^{\int Pdx} = \int Q\ e^{\int Pdx} dx + c$, where $e^{\int Pdx}$ is called the integrating factor (I.F.).

Based on the above information, answer the following questions.(Q1 to Q4)

Q1. The integrating factor of the differential equation sin$x \frac{dy}{dx}$ + 2$ycosx$ = 1 is $(\sin x)^{\lambda}$, where λ =

(a) 0 (b) 1 (c) 2 (d) 3

Feedback

Q.1 diff. eq^n. $\sin x \frac{dy}{dx} + 2y \cos x = 1$ is $(\sin x)^{\lambda}$, where $\lambda = ?$

$\frac{dy}{dx} + 2\cot x y = \operatorname{cosec} x$ $P = 2\cot x$, $Q = \operatorname{cosec} x$

$IF = e^{2\int \cot x\, dx} \Rightarrow e^{2\log \sin x} \Rightarrow e^{\log \sin^2 x}$

$If = \sin^2 x = (\sin x)^{\lambda}$

$\boxed{\lambda = 2}$

Q2. Integrating factor of the differential equation (1− x^2) $\frac{dy}{dx}$ − xy = 1 is

(a) − x (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}$log(1− x^2)

Feedback

② $(1-x^2)\frac{dy}{dx} - xy = 1$

$\frac{dy}{dx} - \frac{xy}{(1-x^2)} = 1$

$P = \frac{-x}{(1-x^2)}$, $Q = 1$

$If = e^{-\int \frac{x}{1-x^2} dx}$ $1-x^2 = t$, $-2x\,dx = dt$, $-x\,dx = \frac{dt}{2}$

$= e^{\frac{1}{2}\int \frac{dt}{t}}$

$= e^{\frac{1}{2}\log t} \Rightarrow \sqrt{t} \Rightarrow \boxed{\sqrt{1-x^2}}$

Q3. General solution of $\frac{dy}{dx}$ + y tan x = sec x is

(a) y sec x = tan x + c (b) y tan x = sec x + c (c) tan x = y tan x + c (d) x sec x = tan y + c

Feedback

(3) General sol. $\frac{dy}{dx} + y\tan x = \sec x$

$P = \tan x$, $Q = \sec x$

$If = e^{\int \tan x\, dx} \Rightarrow e^{\log|\sec x|}$

the general solutions. $\Rightarrow \sec x$

$y\sec x = \int \sec x \cdot \sec x \cdot dx + C$

$y\sec x = \int \sec^2 x\, dx + C$

$\boxed{y\sec x = \tan x + C}$

Q4. The integrating factor of differential equation $\frac{dy}{dx} - 3y = \sin 2x$ is

(a) e^{3x} (b) e^{-2x} (c) e^{-3x} (d) xe^{-3x}

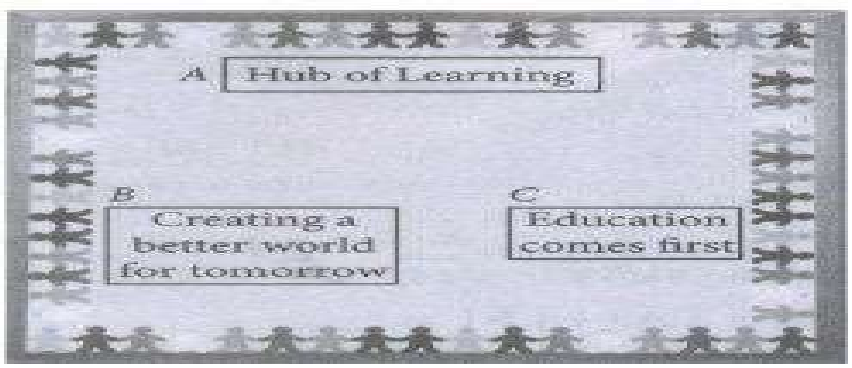
Feedback

(4) $\frac{dy}{dx} - 3y = \sin 2x$

find If. $P = -3$, $Q = \sin 2x$

$If = e^{\int -3\, dx} \Rightarrow \boxed{e^{-3x}}$

Case Study – 2



Three slogans on chart papers are to be placed on a school bulletin board at the points A, B & C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2,2,6) respectively.

Based on the above information, answer the following questions.

Q5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then $\vec{a} + \vec{b} + \vec{c}$ is equal to

(a) $2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ (b) $2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$

(c) $2\hat{\imath} + 8\hat{\jmath} + 3\hat{k}$ (d) $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$

Feedback

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k} + 3\hat{\imath} - 3\hat{\jmath} - 2\hat{k} - 2\hat{\imath} + 2\hat{\jmath} + 6\hat{k} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

Q6.Area of ΔABC is

(a) **19 sq. units** (b) $\sqrt{1937}$ **sq. units**

(c) $\frac{1}{2}\sqrt{1937}$ **sq. units** (d) $\frac{1}{4}\sqrt{1937}$ **sq. units**

Feedback

(6) The vertices of triangle ABC are given as $A(1,4,2)$, $B(3,-3,-2)$ & $C(-2,2,6)$

Adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of ΔABC are given ~~below~~ as:

$$\overrightarrow{AB} = (3-1)\hat{i} + (-3-4)\hat{j} + (-2-2)\hat{k} = 2\hat{i} - 7\hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = (-2-3)\hat{i} + (2+3)\hat{j} + (6+2)\hat{k} = -5\hat{i} + 5\hat{j} + 8\hat{k}$$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -5 & 5 & 8 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = -36\hat{i} + 4\hat{j} - 25\hat{k}$$

$$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \sqrt{1296 + 16 + 625} = \sqrt{1937}$$

(C) $\boxed{\frac{1}{2}\sqrt{1937} \text{ sq units}}$

Q7.Which of the following is not true?

(a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ (b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

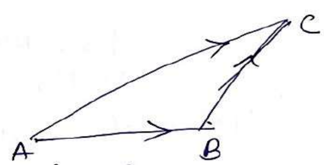
(c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ (d) None of these

Feedback

Q(7)

In ΔABC

$\overrightarrow{AC}$ is the resultant of $\overrightarrow{AB}$ & $\overrightarrow{BC}$.



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ —(1)

Part (A)

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

From eq^n (1)

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} - (-\overrightarrow{CA}) = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ ($\overrightarrow{AC} = -\overrightarrow{CA}$)

True.

Part (B)

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} - (-\overrightarrow{AC}) = \vec{0}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC} = \vec{0}$.

(B) is not True.

Q8.The unit vector in the $\vec{a}+\vec{b}+\vec{c}$ is :

(a) $\frac{2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$ (b) $\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$

(c) $\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$ (d) $\frac{2}{9}\hat{\imath} + \frac{3}{9}\hat{\jmath} + \frac{6}{9}\hat{k}$

<u>Feedback</u>

Q8 unit vector $\cdot$ $\vec{a}+\vec{b}+\vec{c}$ is

$\vec{a}+\vec{b}+\vec{c} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{4+9+36} = \sqrt{49}$

$= 7$

$(\widehat{a+b+c}) = \frac{(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}+\vec{b}+\vec{c}|}$

$= \frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$

$\boxed{c}$

Directions: Each of these questions (Q9 & Q10) contains two statements: Assertion (A) and Reason (R). Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the options (a) , (b) , (c) and (d) given below :

(a) A is true , R is true and R is a correct explanation for A

(b) A is true , R is true and R is not a correct explanation for A

(c) A is true and R is false

(d) A is false and R is true

Q9 Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \frac{dy}{dx} + 5$ are 2 and 1 respectively.

Reason (R): Order of a differential equation is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given differential equation & degree is defined as the power of the highest order derivative.

Feedback

Q (9)

$$\sqrt{\frac{d^2y}{dx^2}} = \left(\frac{dy}{dx}+5\right)$$

squaring both sides.

$$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^2 = \left(\frac{dy}{dx}+5\right)^2$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 + 25 + 10\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 - 10\frac{dy}{dx} - 25 = 0$$

order = 2, degree = 1

A is True, R is True and R is correct explanation. for A

(A)

Q 10. Assertion: Direction cosines of the vector $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ are 2/7, -3/7 , 6/7

Reason: Direction cosines of a vector $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\,\hat{k}$ are a, b, c.

Feedback

Q (10) Vector $2\hat{i}+3\hat{j}-6\hat{k}$ (Assertion)

d' Ratio are $a=2, b=3, c=-6$

d'R cosine are $= \frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Reason: Vector $\vec{r} = a\hat{i}+b\hat{j}+c\hat{k}$.

d'R cosine are $= \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

A is True and R is false.

(C)

Answer Key

Case Study 1	Q1. C	Q2. C	Q3. A	Q4. C
Case Study 2	Q5. A	Q6. C	Q7. B	Q8. C
Q9. A	Q10. C			